Modelling and forecasting brand share: a dynamic demand system approach

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ABSTRACT

This paper proposes a dynamic state-space AIDS model of brand share. The model structure separates the transitory and permanent components of the data and offers several key advantages over conventional share modelling approaches. Firstly, it provides a general framework to examine the time series properties of brand share. Secondly, it provides an accurate assessment of the short-run effects of marketing mechanics. Thirdly, the extracted permanent component can be used in a separate auxiliary analysis of the long-run effects of marketing mechanics. The model is applied to a small segment of the toiletries category comprising four differentiated brands and covering over five years of quad-weekly time series data. The results demonstrate a significant improvement over the conventional AIDS model and indicate short-run competition on the basis of price and promotional activity. Long-run analysis, however, suggests that such activity exerts no sustainable impact on the share of competing brands. Out of sample forecasts are satisfactory and provide an improvement over both a standard dynamic AIDS and time-varying-parameter MNL model. Brand share forecasts, in conjunction with price and segment value forecasts, are then used to produce brand volume forecasts.

Key Words: Almost Ideal Demand System (AIDS), Kalman Filter, non-stationarity, long-run effects, co-integration.

1. Introduction

One of the key features of imperfectly competitive markets is the presence of differentiated products that cater specifically for the heterogeneity of consumer tastes. Identifying substitution patterns between such products in response to changes in price and other marketing variables is central to an understanding and estimation of consumer demand. The varying parameter discrete choice Multi-Nomial Logit (MNL) model of McFadden (1973) represents a popular analytical framework for this purpose. However, owing to an assumption of independent and identically distributed (*i.i.d*) unobserved utilities or error terms, the basic MNL model embodies the Independence of Irrelevant Alternatives (IIA) property. This implies an unrealistic proportional pattern of substitution between competing products, driven solely by market share.

More flexible and realistic substitution patterns can be achieved in several ways. One approach is to allow tastes to vary either randomly or with respect to unobserved variables. This induces correlation between the error terms of each product and dispenses with the IIA feature such that substitution patterns are driven by similarities in characteristics rather than market share. Under these circumstances, however, the error terms are no longer *i.i.d* and the MNL model is a mis-specification of product demand. One solution to this problem is the generalised extreme-value model of McFadden (1978), which provides a structure that does cater for non-*i.i.d* error terms. Alternatively, there is the multivariate probit model demonstrated in the work of Jedidi, Mela and Gupta (1999), or the more advanced approach of the mixed logit model illustrated by Berry (1994), Berry, Levinsohn and Pakes (1995) and Nevo (2000).

An alternative method, which allows for both flexible substitution patterns and the heterogeneity of consumer tastes, is to assume that consumer preferences survive the aggregation process such that a representative consumer exists. This is the approach of the continuous choice Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980a). Many examples of the static AIDS exist and an application to the demand for differentiated goods can be found in Hausman, Leonard and Zona (1994). Dynamic generalisations are also well documented and examples include *inter alia* Blanciforti and Green (1983) and Alessie and Kapteyn (1991). In this paper, we develop the static functional form of the AIDS to include additional marketing mechanics and provide an alternative dynamic generalisation based on state-space modelling techniques.

The proposed model improves on conventional dynamic econometric share models in three key ways. Firstly, the model structure explicitly separates the transitory and permanent components of each brand share. This allows a formal analysis of their time series properties that is statistically superior to the standard unit root tests generally employed in the literature (*inter alia* Dekimpe and Hanssens, 1995, Mela *et al*, 1997). Secondly, the parameters of the marketing variables are directly interpretable as short-run demand effects: parameter estimates in standard short-run models formulated in first differences are not readily interpretable in this way (Hendry, 1995), whereas parameters based on short-run models using lagged dependent variables are biased. Finally, the extracted permanent components can be used to assess the long-run impact of marketing activity in a separate auxiliary model, providing an alternative methodology to existing approaches in the literature (*inter alia* Dekimpe and Hanssens 1995a, 1999, Mela, Gupta and Lehmann, 1997 and Jedidi *et al.*, 1999).

The paper is organised as follows. In section 2, we present the static model. In section 3, we discuss the dynamic generalisation and econometric methodologies. In section 4, we present the data and estimation results. Results are split into statistical analysis, diagnostic checks of the model's data congruence, interpretation of the parameter estimates and an assessment of the long-run impact of price and promotions. In section 5, we present brand share forecasts, together with a time series model for total segment spend. Together, these produce brand volume forecasts. Section 6 concludes with key learnings from the paper and avenues for future research.

2. The static model

Deaton and Muellbauer's Almost Ideal Demand System (AIDS) represents a good framework for analysing market level brand demand. Firstly, the model is derived from the theory of consumer choice, incorporating competitive interaction between differentiated brands and segment level income effects. Secondly, the structure is derived from the Price Independent Generalised Logarithmic preference class (Deaton and Muellbauer 1980b), which allows exact aggregation over households.

The basic linearised AIDS model is written as:

$$s_{it} = \hat{\alpha}_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_{jt} + \beta_i \ln\left[\frac{w_t}{\tilde{p}_t}\right] + \varepsilon_{it} \qquad (1)$$

where w_t = consumer expenditure on the segment at time t, p_{jt} = price of the j^{th} brand at time t, s_{it} = expenditure share of the i^{th} brand at time t = $p_{it}q_{it}/w_t$ and $\tilde{p}_t$ = a group price index defined as $\sum_{j=1}^{n} s_{jt} \ln p_{jt}$. The matrix γ_{ij} measures own and cross-price effects, whereas the vector of β_i coefficients represents a set of (non-linear) Engle curves, measuring the impact on brand i of changes in total segment expenditure. The intercept term $\hat{\alpha}_i$ denotes an autonomous component representing core consumer taste for each brand.[1] Any unexplained variation in brand share is captured by the disturbance term ε_{it}, distributed as IN(0, $\sum_{\varepsilon}^{2}$), where $\sum_{\varepsilon}^{2}$ has rank *n-1* due to the model adding up constraint.

Given that prices are not the sole marketing mechanic used by the manufacturer, the basic model (1) is augmented to include additional marketing variables X_k. This involves a modification of consumer preferences embodied in the underlying AIDS cost function and Dunne, Pashardes and Smith (1984) and Alessie and Kapteyn (1991) provide an example in the context of demographic effects. In this paper, we allow the price coefficients of the cost function to depend on brand specific marketing effects, giving:

$$s_{it} = \hat{\alpha}_{0i} + \sum_{j=1}^{n} \gamma_{ij} \ln p_{jt} + \sum_{k=1}^{m} \sum_{j=1}^{n} \phi_{kij} X_{kt} + \beta_i \ln \left[\frac{w_t}{\tilde{p}_t} \right] + \varepsilon_{it} \tag{2}$$

where ϕ_{kij} represents the matrix of own and cross-effects for the k^{th} variable X_k at time t.

3. The dynamic model and econometric methodologies

As it stands, the model defined by equation (2) is static. The conventional approach to incorporating dynamics starts by treating the static relationship as synonymous with a long-run or equilibrium allocation. Under these circumstances, equation (2) implicitly assumes that the representative consumer instantaneously adjusts to a new equilibrium in response to changes in the explanatory variables. Factors such as brand loyalty and habit formation, however, suggest that this is unlikely and it is preferable to formulate a short-

[1] Brand intercepts are interpreted as measures of brand health or equity in the discrete choice modelling literature (*inter alia* Kamakura and Russell 1993, Jedidi *et al.* 1999). In the AIDS, the intercepts may be used as relative measures of brand equity.

run equation capturing the dynamics of sluggish adjustment to the new equilibrium. Alessie and Kapteyn (1991) provide a good example, where the taste parameters embodied in the intercepts of the AIDS are expressed as a linear function of lagged budget shares. Current preferences then depend on past consumption patterns and (2) may be reformulated to give a short-run adjustment model. A more sophisticated approach is to treat equation (2) as a set of co-integrating relationships between non-stationary variables (Engle and Granger, 1987).[2] Conventional OLS estimation then provides consistent estimates of the long-run parameters and dynamic vector error correction models can be estimated, incorporating long-run information from the co-integrating solutions. Sluggish adjustment is often motivated by adjustment costs and an application can be found in Barr and Cuthbertson (1991).

In this paper we adopt an alternative dynamic generalisation of the AIDS, which allows tastes to change over time in the light of new information. Equation (2) is no longer viewed as an equilibrium relationship towards which the system gradually adjusts: rather, it is a relationship which holds only locally in time and subject to shifts as tastes evolve. We focus solely on the taste parameters embodied in the intercepts of the AIDS, which are expressed as a stochastic function of time, thereby introducing a Bayesian evolution process as documented in West and Harrison (1997).

The proposed taste evolution is introduced using the structural time series methodology of Harvey (1989), and applied examples can be found in *inter alia* Hunt, Judge and Ninomiya (1999) and Moosa and Baxter (2002). Equation (2) is re-written thus:

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[2] Co-integration between non-stationary variables implies the presence of a common stochastic trend and a stationary long-run equilibrium relationship between them. Any long-run analysis should incorporate such potential relationships between integrated variables. This concept plays an important role in developing the auxiliary long-run model below and assessing long-run substitutability between brands in section 4.

$$s_{it} = \mu_{it} + \delta_{it} + \sum_{j=1}^{n} \gamma_{ij} \ln p_{jt} + \sum_{k=1}^{m} \sum_{j=1}^{n} \phi_{kij} X_{kt} + \beta_i \ln \left[\frac{w_t}{\tilde{p}_t} \right] + \varepsilon_{it} \tag{2a}$$

$$\mu_{it} = \mu_{it-1} + \lambda_{it-1} + \eta_{it} \tag{3a}$$

$$\lambda_{it} = \lambda_{it-1} + \xi_{it} \tag{3b}$$

$$\delta_{it} = -\sum_{j=1}^{p-1} \delta_{it-j} + \kappa_{it} \tag{3c}$$

Equation (2a) replaces the intercept $\hat{\alpha}_{0i}$ in equation (2) with a stochastic trend μ_{it}, comprising two components described by equations 3(a) and (b). Specifically, equation (3a) allows the intercept level for each share to follow a random walk with a growth factor λ_i. Equation (3b) allows λ_i itself to follow a random walk. The variables η_{it} and ξ_{it} represent two mutually uncorrelated normally distributed white-noise error vectors with zero means and covariance matrices $\Sigma_\eta^2 \text{and} \Sigma_\xi^2$. Various dynamic structures are encompassed within this general specification. For example, if both covariance matrices are non-zero, the share levels follow a random walk with stochastic drift. If $\Sigma_\eta^2 \neq 0$ and $\Sigma_\xi^2 = 0$, the drift component is deterministic. If $\Sigma_\eta^2 \neq 0$ and the growth factor is zero then the levels follow a random walk without drift. If both covariance matrices are zero, the shares are trend stationary. In this way, the system can accomodate both stationary and non-stationary market shares and allows the data to decide between the two. Equation 3(c) specifies seasonal effects, which are constrained to sum to zero over any one year and thus cannot be confounded with other model components.[3] Stochastic seasonality is allowed for using dummy variables, where *p* denotes the number of seasons per year, δ_t is

[3] Seasonality is often overlooked in both static and dynamic forms of the AIDS. However, Hausman and Leonard (2002) do incorporate deterministic seasonality in an AIDS model of the bath tissue industry.

the seasonal factor corresponding to time t and κ_t is a random error with mean 0 and covariance matrix σ_κ^2. If the latter is zero, then seasonality is deterministic.

The proposed dynamic formulation improves on the conventional approach for three reasons. Firstly, each brand share is decomposed into permanent and transitory components, thus providing a robust framework in which to analyse their time series properties. For example, with $\Sigma_\varepsilon^2, \Sigma_\eta^2 \text{ and } \Sigma_\xi^2 \neq 0$, the model has an Autoregressive Integrated Moving Average (ARIMA) (0,2,2) reduced form (Harvey, 1989). If accepted by the data, this corresponds to a dynamic regression of the second difference of brand share plus explanatory variables with a second order MA error term. Brand share behaviour is thus split into a non-stationary I(2) component capturing the long-run evolution and a stationary MA(2) component capturing the short-run dynamics.[4] The time series properties of market share are usually assessed via the autoregressive based unit root test of Dickey and Fuller (1981). Examples include Lal and Padmanabhan (1995) and Dekimpe and Hanssens (1995b). However, such tests have low power against a deterministic trend alternative and ignore the moving average structure of the data, thus displaying poor statistical properties (Schwert, 1985). The richer dynamic structure of the proposed model thus allows a more robust test of the unit root hypothesis.

Secondly, the parameter estimates of the marketing variables are directly interpretable as short-term effects. Specifically, the μ_{it} series of equation 3(a), if supported by the data, measure permanent changes in brand demand arising through shifts in underlying

[4] This extracts the permanent component(s) of non-stationary time series and provides an alternative to the persistence measures advocated by Campbell and Mankiw (1987) and Dekimpe and Hanssens (1995a).

consumer tastes and absorb any long-term marketing effects.[5] This leaves the γ_{ij}, ϕ_{ij} and β_i parameters of equation 2(a) to measure short-run changes due to current period marketing activity. Conventional short-run dynamic models involve lagged shares or error correction formulations. The former introduce parameter bias due to the presence of lagged dependent variables and are unsuitable for non-stationary data. The latter do not produce readily interpretable short-run parameter estimates due to first differencing of the data. (Hendry, 1995). The proposed approach avoids both problems.

Finally, the extracted trend components μ_{it} can be used to assess the long-run impact of marketing activity. This feature is explored with the following auxiliary model:

$$\Delta\mu_{it} = \pi_{it} + \sum_{k=1}^{m}\sum_{j=1}^{n}\psi_{ij} z_{kjt} + u_t \qquad (4a)$$

$$\pi_{it} = \pi_{it-1} + \rho_{it} \qquad (4b)$$

where $\Delta\mu_{it}$ represents a vector comprising the non-stationary first difference (Δ) of each extracted trend for brand share i and z_{kjt} represents a matrix of k non-stationary marketing "pressure" variables.[6] The variable π_{it} represents an intercept, analogous to that in 2(a), which is allowed to evolve according to 4(b). If the non-stationary behaviour of the first differenced trend series can be explained by the non-stationary behaviour of the marketing pressure variables, then a common trend exists and the variables co-integrate. This may be tested following the arguments of McCabe, Leybourne and Shin (1997),

[5] Similar points are made by Moosa and Baxter (2002). The μ_{it} series also absorbs shifts in consumer tastes due to evolution in the marketing parameters and the effects of explanatory variables omitted from equation 2(a). We return to this issue in an empirical application of the auxiliary long-run model in section 4 below.

[6] The extracted trend component μ_{it} is, data permitting, a non-stationary I(2) variable. The first difference is a non-stationary I(1) variable by definition, with stochastic trend given by the drift component λ_i. Marketing pressure variables can be created as the integrals of the basic marketing variables, representing the cumulative build over the sample. Operalization of these variables is detailed in section 4 below.

where under the null hypothesis of co-integration the variance of the level disturbance ρ_{it} is zero and the intercept collapses to a constant. A negative (positive) co-integrating relationship implies that marketing innovations permanently decrease (increase) the trend growth rates. Consequently, even in the event that marketing activity returns to prior values, brand shares remain at permanently lower (higher) levels.[7]

Model 4 provides a time series methodology for assessing the hysteresis effects of marketing (Little, 1979) and improves on current approaches in the marketing literature. Firstly, conventional time series approaches, as demonstrated in the work of Dekimpe and Hanssens (1995a, 1999) and Nijs, Dekimpe, Steenkamp and Hanssens (2001), involve either VAR or VECM models and standard unit root tests. The problems with the VAR methodology are outlined in Harvey (1997), whereas the standard unit root test is subject to the statistical problems detailed earlier. Secondly, varying parameter techniques as demonstrated in the work of Mela *et al.* (1997) and Jedidi *et al*. (1999) do not identify genuine long-run (permanent) effects. For example, Mela *et al.* also use an auxiliary regression approach involving non-stationary I(1) data. However, in the absence of co-integration between the variables, such regressions are spurious (Granger and Newbold, 1974). Jedidi *et al*. on the other hand introduce parameters that vary as a function of stationary long-term pressure marketing variables. Consequently, the parameters are not truly evolving *per se* and long-run effects are not permanent. The proposed approach, however, uses genuine time-varying (non-stationary) parameter series and is not subject to spurious regression problems.

[7] The more conventional Engle-Granger (1987) or Johansen (1988) techniques may also be used to test for co-integration between the variables in 4(a). However, these rely on autoregressive based testing using the Dickey-Fuller (DF) or Augmented Dickey Fuller (ADF) approach. The proposed methodology is more in the spirit of the structural time series approach of model 2.

Equations 2(a)-3(c) and 4(a)-(b) provide the econometric framework of this paper. Before proceeding to an empirical application, it is worthwhile highlighting two key limitations. Firstly, we focus solely on evolution in the intercepts of equation 2(a) and do not directly model evolution in the marketing parameters. Consequently, we do not examine the long-run impact of promotions on price elasticities (Mela *et al.* 1997, Jedidi *et al.* 1999) or consumer stockpiling (*inter alia* Foekens, Leeflang and Wittink, 1999).[8] Secondly, the auxiliary long-run model assesses trend evolution exogenously to the dynamic model and as such, is relatively *ad hoc*. An alternative technique, demonstrated in Smith, Hall and Mabey (1995) in the context of energy demand, is to incorporate the relevant regressor variables directly into 3(b), giving a self-contained dynamic model with endogenous trend evolution. Both limitations are, however, fully justifiable. Introducing evolution into each of the marketing parameters substantially reduces degrees of freedom: there are 8 marketing variables, 1 income variable and 12 seasonal dummies in each equation of 2(a). Consequently, together with the level, slope and seasonal variances, the model is already quite heavily parameterised. Trend evolution is assessed exogenously since the stochastic processes specified by equations 3(a) and (b) are designed to separate the permanent and transitory component(s) of brand share, central to the stationarity testing and empirical analysis of section 4 below. This would not be possible if deterministic marketing components were incorporated directly into the trend evolution process. Consequently, the endogenous trend approach should be reserved for a model designed to look specifically at the long-run effects of marketing and represents a fruitful area for future research.

[8] Although evolution in the marketing parameters is not explicitly modelled, it is not completely excluded from the model. Any such evolution is indicative of shifts in brand equity as consumer sensitivity to price and promotions changes and, as noted earlier, is absorbed into the stochastic trend series.

Each model constitutes a separate Seemingly Unrelated Time Series Equation (SUTSE) model - the time series analogue of the SURE model. Dependent variables are linked through the correlations of the disturbances driving the unobserved components. Estimation proceeds using the modelling procedure described by Koopman, Harvey, Doornik and Shephard, (2000), which sets the models up in state-space form. For the main model 2(a)-3(c), this relates the observable brand shares s_{it} to the unobservable state variables μ_{it}, λ_{it}, δ_{it}, via the measurement equation 2(a). For the auxiliary model 4(a)-(b), this relates the extracted trend growth rates to the unobservable state variable π_{it}. Both models are then estimated separately by maximum likelihood, using a Kalman Filter to update the state vector as new observations become available.

4. Data and estimation results

i) The dynamic model

To demonstrate the proposed modelling approach, we analyse a small segment of the toiletries category comprising four differentiated brands, covering over five years of quad-weekly time series data from 1995(5) to 2000(12). All data are taken from the IRI Infoscan database and aggregated over stores. Brand expenditure shares equal the ratio of brand sales to total segment spend, whereas brand prices are calculated as the ratio of brand value sales to brand volume sales. The X_k matrix in equation 2(a) contains indices of promotional activity for each brand, equal to the percentage of base volume on any deal.[9] To simplify estimation, we assume that price and promotions are exogenous.

[9] This variable is an average of several different types of deal such as in-store display and multi-buys. It is weighted for the promoted product's share of trade in-store and represents the only additional marketing mechanic. Advertising data are not available since they have to be purchased at considerable cost. Basic data descriptives for all variables are given in Table 7 in the Appendix.

Furthermore, we also assume that segment expenditure is exogenous. It is certainly possible that brand share is determined simultaneously with segment value. However, we follow Hausman *et al* (1994) and assume that higher-level economic change dominates, where segment expenditure is driven by changes in the wider economic environment.

a) Statistical analysis: trends and long-run substitutability

The behaviour of the brand shares over the sample is shown in Figure 1 in Appendix 1. Certain stylised facts are immediately apparent. Firstly, the segment is divided into two groups, with Brands A and B securing the bulk of the market. Secondly, all data appear to exhibit local trending behaviour with two clear phases for both groups: pre and post-1999 for A and B and pre and post-1998 for C and D. Thirdly, Brands A and B tend to mirror each other as do Brands C and D, suggesting substitutability between them. Applying model 2(a)-3(c) gives the end-of-state results in Table 1 of the appendix, with t-statistics in parenthesis. Model test statistics are given in Table 2, compared to two rival specifications. Model 1 is the stationary form of model 2(a): that is, OLS estimation with deterministic trend and seasonality. Model 2 is the non-stationary dynamic form with deterministic seasonality. Model 3 is the estimated model with full stochastic dynamics. Model 1 is clearly unsatisfactory, with autocorrelation in all equations. Furthermore, the seasonal R^2 for Brand D is negative implying that the fit for this equation is worse than a simple random walk with drift model (Harvey, 1989). Model 2 improves matters, with all seasonal R^2 statistics now positive and a substantial reduction in autocorrelation. However, problems remain for Brand C, with a deterioration in normality for Brands A and B. Model 3 removes both problems. Given the fairly heavy parameterisation of Model 3, we next consider the Akaike information criterion, which allows a valid statistical comparison of rival models with different numbers of parameters. We use the ratio of the criteria for Models 1 and 2 to the criterion for Model 3, where a reduction in the criterion indicates an improvement in model fit. All ratios are greater than unity indicating that Model 3 is superior. Finally, in order to test whether the differences between the three models are statistically significant, likelihood ratio statistics are computed comparing the full stochastic specification with the two (nested) restricted alternatives. Test statistics are distributed as $\chi^2(r)$, where r denotes the number of restrictions: these are $\Sigma_k^2, \Sigma_\eta^2$ and Σ_ξ^2 = 0 for model 1 and Σ_k^2 = 0 for model 2. Results are 121.2 and 6.2 respectively. With $\chi^2(3)$ and $\chi^2(1)$ critical values of 7.8 and 3.8 at the 5% level, all restrictions are unacceptable, indicating that Model 3 is statistically superior.

The above diagnostic results clearly indicate that the share data for this segment exhibit non-stationary trend and seasonal behaviour. Figure 2 plots the extracted trends, which each contain two unit roots and represent the long-run evolution of consumer tastes.[10] The first difference of these series represent the trend growth rates, with an underlying stochastic drift given by the slope components λ_i plotted in Figure 3. The long-run substitutability between A and B is clearly visible in the trend plots, echoing the pattern in the raw data. That between C and D is less so. Such graphical evidence is corroborated by the covariance of the level disturbance given in Table 3.[11] This is a 3x3 matrix since we are estimating n-1 share equations, with Brand D residually determined via the model's adding up constraint. The upper triangular part provides the correlations between

[10] Since the extracted trends absorb shifts in parameters of the marketing variables in equation 2(a), as well as the effects of omitted explanatory variables, the non-stationary evolution of consumer tastes can sometimes take on an erratic appearance.

[11] None of the disturbance variances are zero. Restricting the level co-variance matrix to zero is rejected, providing further evidence of non-stationarity. Details are provided in Table 3 of the appendix. This contrasts with standard (ADF) unit root tests, which indicate stationarity for brand shares A-C.

the level components, where we have high negative correlation (-0.9) between Brands A and B, yet less (-0.62) between A and C. This suggests (inverse) relationships between the underlying trend components, indicative of long-run substitutability.

In order to formally test for this, we use the factor-load matrix, which is equivalent to a Cholesky decomposition of the covariance matrix (Koopman *et al.* 2000). Long-run substitutability between brands implies an (inverse) equilibrium relationship between the underlying trends: this is so if the level components co-integrate and move inversely together over time. Co-integration between the level components implies a level co-variance matrix - and corresponding factor-load matrix - of reduced rank: that is, common factors exist between the brand share level components.[12] With 3 shares, there can be, at most, 2 common factors and thus 1 co-integrating vector. To simplify matters, we restrict this to one common factor and proceed by reducing the rank of the co-variance matrix from three to one and re-estimate the model. The unrestricted (full rank) level factor-load matrix given in Table 3 contains 3 columns, the first of which is (1, -0.8, -0.24)′. The restricted (reduced rank) matrix contains only one column, given as (1, -1.1, -0.1). A likelihood ratio test statistic of 5.1 against a $\chi^2(2)$ critical value of 5.6 at the 95% level demonstrates that this is an acceptable restriction. Consequently, a single long-run equilibrium relationship does exist between Brands A and B and C: the adding up constraint implies that this equilibrium relationship also covers Brand D.

Having established the existence of a common level factor, restrictions on the elements of the factor-load matrix can now be applied to test the degree of long-run substitutability

[12] Co-integration implies an equilibrium relationship in the same sense as in Footnote 2 and Section 3 above. Here, however we are interested in long-run relationships between the brand shares. Definitions of the factor load matrix and Cholesky decomposition are given below Table 3 in the appendix.

between the brands. The load coefficient for Brand B is negative and particularly high, suggesting a high degree of substitutability between A and B: restricting the second element of the one remaining column of the level factor-load matrix to –1 and re-estimating the model gives a likelihood ratio test statistic of 1.04 against a $\chi^2(1)$ critical value of 3.8. Thus, we cannot reject the null of perfect negative correlation in the levels: Brands A and B can be viewed as perfect long-run substitutes. The load coefficient for C is very low suggesting little long-run substitutability: restricting the third element of the one remaining column of the level factor-load matrix to zero and re-estimating the model gives a likelihood ratio test statistic of 0.95. Consequently there is no evidence of long-run substitutability between A and C, or between B and C. Since the model adding up constraint imposes a zero column sum condition on the coefficients of the factor load matrix, these restrictions also imply that the load coefficient for D is zero; that is, there is no evidence of long-run substitutability between D and any other brand. This is consistent with the weak mirroring between C and D in Figure 2, implying that the apparent substitution between these brands in Figure 1 is predominantly short-term.[13]

[13] Testing illustrates that we cannot reject the null of common growth rate trends between the brands, echoing the plots in Figure 2. However, this tells us nothing about long-run substitutability: without co-integration in the levels of each series, growth rates will not move (inversely) together over time.

b) Economic interpretation of parameter estimates

i) Price effects

The conditional own and cross-price demand elasticities are derived from the coefficient estimates of Table 1. The results and formulae used are given in Table 4.[14] All own price elasticities are negative, elastic and significant at the 10% level for Brands A, B and D and at the 5% level for C. These results have important implications for pricing policy. The managers of these brands can, *ceteris paribus*, discount average price in response to a fall in production costs and expect a revenue increase. On the other hand, they are unable to pass cost increases onto the consumer and thus raise average price without incurring a revenue decrease.

Competitive cross-price elasticities are derived from the off-diagonal cross-price terms. Firstly, there is no evidence to support any price substitution between Brands A and B. Statistical insignificance here suggests that, on average, price is not a particularly effective tactical marketing mechanic in the competitive activity between these two brands. Secondly, there is evidence to suggest that C is a strong price substitute for Brand D. The latter is, therefore, clearly vulnerable with Brand C as a player in the market. Yet, the reverse is not true implying that Brand C's pricing policy is more effective than D's.[15]

[14] Price elasticities involve both the price and income coefficient estimates. Consequently, we cannot use the t-ratios associated with the price coefficients to determine statistical significance. Rather, we must use significance levels relating to the distribution of the elasticities. These are calculated using a bootstrapping technique, where we find that all own elasticities are significant. Details are provided in the appendix. Price elasticities are also conditional on constant total segment expenditure. Unconditional elasticities can be derived by incorporating the elasticity of the segment with respect to an aggregate segment price index (Hausman *et al.* 1994). This would require a complete hierarchical model of the toiletries market.

[15] The AIDS demand equations provide first order approximations to any set of demand functions, with perfectly viable elasticity estimates. Consequently, for simplicity, we have not imposed a symmetric price matrix. If, however, we wish to use the results to pursue a welfare analysis of price changes then consistency with consumer theory is strictly necessary and a symmetric price matrix is required.

Finally, there is evidence that Brand C is a weak substitute for Brand A: a 1% increase in the price of C leads to a 0.2% increase in demand for A. This implies that A must take into account the pricing decisions of C when setting its own short-run policy, providing further evidence of the relative effectiveness of C's pricing stance in the segment. On the other hand, we find that A is a weak substitute for D, implying that the latter must take into account the pricing decisions of the former when setting its own policy.

The overall conclusion is that the short-run pricing policy of Brand C is the most effective in the segment, with significant steal from A and D. In view of its small market share, this firmly establishes its role as a strong fringe price competitor. However, there is no potential for C's pricing tactic to generate a sustainable competitive advantage with respect to Brands A and D over the longer term. This follows from the earlier common factor analysis, together with the plots of Figure 2, which indicate systematic and sustainable long-run brand switching between Brands A and B alone. The implication is that the market share of Brand C relative to A and D is only temporarily affected by its pricing activity. Consequently, the Brand C manager may cut average price to drive short-term incremental volume, but cannot rely on this to maintain permanent shifts in market share. This insight clearly highlights the benefit of a model capable of separating long-run movements in the underlying brand share level from the short-run dynamics.

ii) Promotional effects

The conditional own and cross-promotional demand elasticities are derived from the coefficient estimates of Table 1 using the formulae provided in the appendix. The results are given in Table 4. We would expect positive diagonal own effects, as promotional activity boosts demand. All are positive save that for Brand C, which is insignificant at both the 5% and 10% level. The own promotional effects for Brands A and B are significant at the 5% level and inelastic. Thus, a 1% increase in the promotional activity of Brand A leads to a 0.06% increase in consumer demand in the short-term - drawn from competing Brands B, C and D. Conversely, a 1% increase in promotional activity of Brand B leads to a 0.04% increase in demand - drawn from competing Brands A and C.[16] The off-diagonal terms represent the competitive effects of promotional activity. There is evidence of strong steal between Brands A and B implying that both are substitute brands. Compared with the price elasticity results, it is clear that both compete more on the basis of promotions in the short-run. Significant cross-effects suggest that Brand C must take into account the actions of both Brands A and B when setting its own policy. However, the reverse is not true. The promotional activity of C has no significant effect on A and B, further demonstrating the more effective promotional activity of these two brands. Brand D's promotions are successful in drawing funds from C implying that the former is a weak substitute for the latter. However, the reverse is not true. This contrasts with the competitive price interaction seen earlier and implies that D's promotions are more effective than price with respect to C. Interestingly, an increase in the promotional activity of Brand D tends to raise category awareness to the extent that funds are diverted into additional purchase of Brand A.[17] The dominant position of A with respect to D is further reinforced by the former's successful promotional steal, implying that A is a substitute for D matching the price effects.

[16] Elasticity estimates are small, possibly due to significant cross effects and aggregation across time and types of deal. The aggregation bias identified by Christen *et al.* (1997) is less of a problem due to a linearised functional form and use of percentage of base volume on deal. This controls for store size and is more suited to aggregate analysis than a conventional percentage of stores on deal promotional measure.

[17] Positive cross-effects are possible since promotional activity can help to raise awareness of all brands in the segment. A similar positive "halo" effect for Brand D comes from Brand B's activity.

The overall conclusion is that Brand A is the strongest promotional player, whereas, in contrast to the price effects, Brand C is the weakest with no steal from any other brand. Again, there is a clear distinction between short and long-run substitution patterns, highlighting promotional policies that enjoy short-term competitive success but cannot contribute to long term sustainable movements in own or competitor market share.

ii) The long-run effects of price and promotions

The results of the previous section have demonstrated a broad pattern of short-run competition compared to the statistical evidence of section 4(a), where taste evolution illustrated long-term switching between Brands A and B only. In the first place, this precludes any long-run sustainable shifts between Brands C and D and implies that despite any short-run success, the marketing tactics of these brands can play no part in gaining a long-run competitive advantage. Secondly, it highlights that Brands A(B) need to focus their marketing effort upon influencing consumer tastes for B(A) in order to generate long-run market share growth. The next step is to assess the extent to which price and promotional activity can help – or hinder – the achievement of this goal.

The separate auxiliary model (4) is used for this purpose, where the cumulative sum from *t* to *t-n* of the short-term price and promotions variables used in equation 2(a) give I(1) series of marketing activity, which serve as proxies for long-term "pressure" analogous to those used in the work of Jedidi *et al* (1999).[18] The integral of the short-term promotional

[18] Provided that marketing variables are stationary, this approach uses a zero decay parameter and generates non-stationary pressure variables by definition. The use of non-zero decay parameters, as in Jedidi *et al*, creates necessarily stationary series and thus cannot provide valid co-integrating relationships. Univariate local-level structural models for each price and promotional variable give zero variances for the level factors, indicative of stationary variables.

data traces out the evolution over the sample, whereas the integral of each brand price represents the cumulative sum of period-to-period price changes. The share constraint of model 2(a)-3(c) implies that $\sum_{i=1}^{n} \Delta\mu_{it} = 0$. Consequently, to avoid linear dependency, equation 4(a) is estimated for n-1 trend growth series with the n^{th} residually determined. The dimension of x_{kjt} depends on the long-run substitutability between brands. For long-run substitutes, trend movements in one will be systematically mirrored by (inverse) movements in the other(s) as expenditure flows between them. If, on the other hand, brands are not long-run substitutes there is no systematic sustainable flow of expenditure between them and the marketing activity of one will not exert any lasting impact on the other(s).[19] Long-run behaviour of these brands may, however, be driven by expenditure remaining from the interaction between long-run substitute brands. These considerations imply that (4a) need only be specified for each $\Delta\mu_{it}$ in terms of price and promotions for long-run substitute brands. The results are presented in Table 5.

Heteroscedasticity and normality diagnostics appear satisfactory, yet R^2 is poor for all equations with residual correlation for $\Delta\mu_{Ct}$ and $\Delta\mu_{Dt}$. Parameter estimates indicate that an increase in the long-term pressure of own-promotional activity has a negative impact on brand tastes, echoing the conventional wisdom that excessive promotional activity erodes brand equity (*inter alia* Jedidi *et al.*, 1999). Cross-promotional activity for Brand A and B indicates long-term brand switching as a result. Price parameter estimates indicate that increasing prices have a negative influence on brand equity, suggesting that as average price escalates, even loyal consumers switch out of the brand. However, for

[19] The AIDS model, from which the trend components are derived, implies that all brands are either substitutes or complements in terms of short-run pricing and marketing activity. However, this does not guarantee permanent or lasting preference shifts between brands indicative of long-run substitutability.

such relationships between I(1) variables to be statistically valid we require acceptance of the null hypothesis of co-integration, which requires that each level disturbance variance σ_{ρ}^{2} is zero. Table 5 illustrates that this is not so, indicating the absence of a common trend. Consequently, we can reject the null of co-integration and thus fail to find a valid long-run impact of price and promotional activity on core tastes for any brand.[20]

This result can be viewed in one of two ways. We may take it at face value: price and promotions have no impact on brand equity, reflecting the notion that the relative position of players in the market is only temporarily affected by marketing activity (Lal and Padmanabhan, 1994). However, this is unlikely as the plots of Figure 2 are indicative of long-run competitive activity between the two brands. A more plausible explanation, is that equation 4(a) is mis-specified. The trend component of model 2(a) is specified as a general stochastic process, potentially absorbing the influence of many other variables. Advertising, economic and demographic fluctuations for example may also impact the evolution in consumer tastes and required to form a co-integrating relationship. These considerations suggest that an improved specification of the taste evolution process is required if the effects of marketing activity on brand equity are to be correctly identified. It is hoped that the material in this section stimulates additional research in this direction.

[20] The absence of co-integration implies that any significant price and promotional effects are spurious and simply the result of regressions involving I(1) data (Granger and Newbold, 1974).

5. Forecasting brand share and brand volume

i) Brand share forecasts

Brand share forecasts are constructed by estimating the model up to 2000(2) and extrapolating the end-of-state level and slope estimates, combined with the parameter estimates and future values of the explanatory variables. Results are presented in Figure 4 of the appendix. Average errors over the period are 2.6%, 3.8%, 6.1% and 9.8% for Brands A, B, C and D respectively, where it is interesting to note that the two smaller brands provide the poorest forecasts. In order to demonstrate relative forecast accuracy, we compare these results with those from two alternative model structures. Firstly, we estimate a conventional dynamic AIDS model, such as that presented in Alessie and Kapteyn (1991), incorporating own and cross-equation lagged shares and a deterministic seasonal component. Forecast errors of 12.3%, 11.2%, 8.9% and 9.8% indicate that failure to acknowledge the non-stationary behaviour and moving average components can lead to inferior forecasts. Secondly, we estimate a time-varying-parameter cross-effects attraction model with stochastic seasonality.[21] Although an improvement over the conventional dynamic AIDS, the non-linear discrete choice structure generates inferior forecasts to those from the dynamic linearised AIDS put forward in this paper, with average errors of 3.0%, 6.5%, 11.9% and 24.8% for Brand A, B, C and D respectively. This highlights that a continuous choice structure, where consumers purchase a mix of heterogeneous brands, is perhaps more appropriate for this segment. Furthermore, it

[21]The attraction model is equivalent to an aggregate form of the MNL model conventionally applied to individual choice data. Introducing cross effects deals with the problematic IIA assumption. However, this form of the MNL model cannot be derived from the consumer's indirect utility function. The dependent variable in the MNL model form is a Log Ratio Share. These are log shares, defined relative to a base log share *n*. Consequently, $LRS_i = LS_i\text{-}LS_n$. After estimation, the n^{th} share may be calculated as $S_n = 1 \big/ \Sigma_i((\exp LRS_i) + 1)$ and the remaining shares as $S_i = (\exp LRS_i) * S_n$.

demonstrates the importance of incorporating income effects, which categorise brands into luxury and necessity goods.[22] To formally assess the forecasting performance of the proposed model against the conventional dynamic and attraction counterparts, we use the test developed by Harvey, Leybourne and Newbold (1997). The results are given in Table 6. With a critical value of 1.75 for a one-sided t-test at the 5% level we can reject the null hypothesis of equivalent forecasting power for each equation of the proposed model.

ii) Brand volume forecasts

Forecasts of brand volume require forecasts of total segment demand. In the brand share model, segment value was exogenously determined, driven by category and economic change. Consequently, segment value W_t is modelled as a univariate local linear trend model with seasonality, which is assumed to approximate higher-level influences on segment demand. In the spirit of the main brand share model, the underlying level is allowed to stochastically trend with stochastic drift. Stochastic seasonality is also allowed. Combining the brand share forecasts with the total segment forecasts and deflating by brand prices gives average percentage errors of 5.8%, 6.6%, 10.5% and 13.2% for Brands A-D.

[22] Estimated expenditure elasticities are given in Table 4 of the appendix.

6. Conclusions

This paper has incorporated two taste-shifting factors into the conventional AIDS model of consumer choice, introducing brand specific promotional activity and allowing the taste parameters embodied in the intercepts to evolve over time. These modifications re-cast the AIDS as a multivariate structural time series model with additional explanatory variables. The model was then applied to the market share of four differentiated brands in a small segment of the toiletries category. The model performed well in terms of statistical diagnostics and was clearly superior to two rival model specifications. This highlighted the evolving brand tastes and seasonality inherent in the sample market shares. Results demonstrated clear long-run substitutability between Brands A and B, contrasting sharply with a much broader pattern of short-run competition between all component brands on the basis of price and promotional activity. Subsequent co-integration analysis using the extracted trend components, however, indicated no significant effect of price or promotional activity on underlying brand tastes. This may well have been indicative of a mis-specified long-run model, where other variables such as advertising may be required to form valid co-integrating relationships.

Comparing model forecasts with those from a standard dynamic AIDS model indicated that the failure to acknowledge both the (non-stationary) autoregressive and (stationary) moving average elements in the data can lead to inferior forecasts. Furthermore, inferior forecasts from an aggregate MNL (attraction) model indicated that the continuous choice structure can outperform the discrete choice structure. Combining our model forecasts with those of the exogenous budget constraint provided brand volume forecasts. Despite the lack of advertising as a key driving variable, the results were eminently reasonable.

These learnings highlight the three key contributions of this paper. Firstly, the model structure provides a convenient method of separating the short and long-run behaviour of brand market share, thereby allowing a formal analysis of their time series properties. Secondly, the model avoids unit root testing and first differencing of the data - providing marketing variable parameters that are directly interpretable as short run own and cross effects describing short-run substitution patterns between component brands. Thirdly, we have demonstrated how the model's extracted trend component can be used to describe the pattern of long-run substitutability in the system. This, in conjunction with co-integration analysis widely used in the economics literature, provides a useful methodology for assessing the long-run effects of marketing mechanics. Two avenues for future research emerge from these issues.

The assumption of exogenous segment spend could be relaxed, thus admitting the possibility that marketing strategy can determine brand share simultaneously with total segment value. This explicitly acknowledges that marketing is not a zero-sum game and segment spend should be treated as an endogenous variable in the system. Not only would this potentially provide more accurate parameter estimates and forecasts, but would also allow us to gauge the extent to which marketing mechanics can help grow the segment. The model would then be more suitable for an analysis of retailer category price and promotion decisions as documented in Anderson and Vilcassim (2001).

More work is required on the long-term effects of marketing activity. Additional determinants of taste evolution, such as advertising, could be incorporated into model (4).

Alternatively, the structure of the dynamic model could be modified to incorporate marketing variables directly into the trend evolution process. Furthermore, evolution in the marketing parameters may be explicitly incorporated into the dynamic model via a straightforward augmentation of the model state vector. This may then be used to explore additional issues in the literature raised by Mela *et al.* (1997) and Jedidi *et al.* (1999). For example, by allowing the price parameters to evolve in this way, it is possible to assess the extent to which price elasticities are influenced by long-term pressure in advertising and promotions, either exogenously using the co-integration methodology of section 3 or endogenously within the model structure.

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Figure 1: Basic Share Data

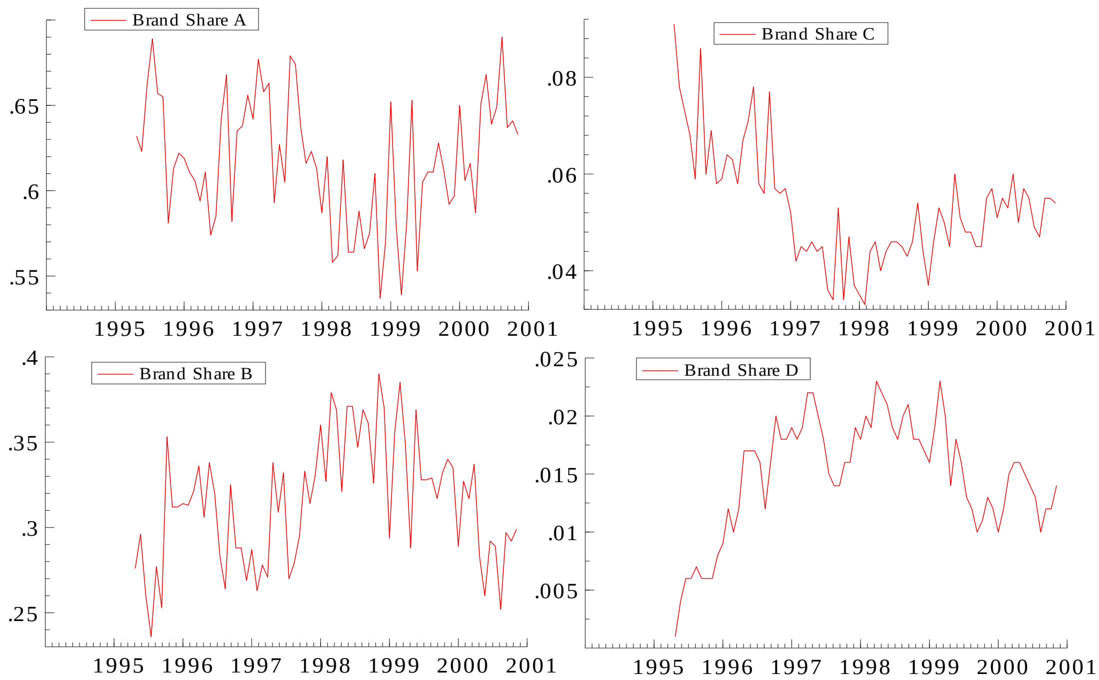


Figure 2: Extracted Share Trends

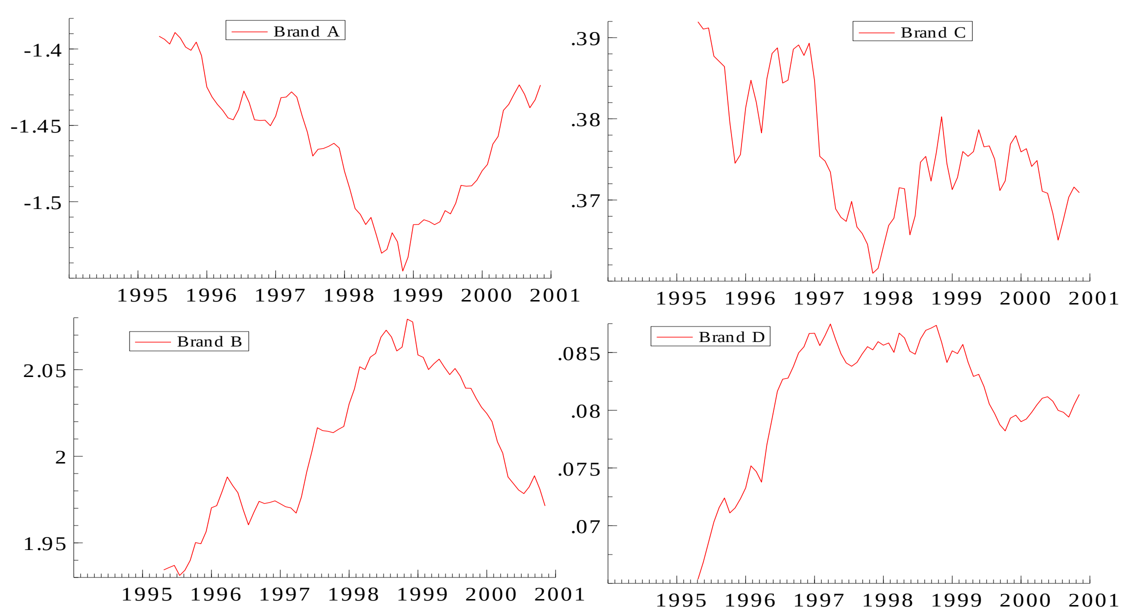


Figure 3: Extracted Slope Components[23]

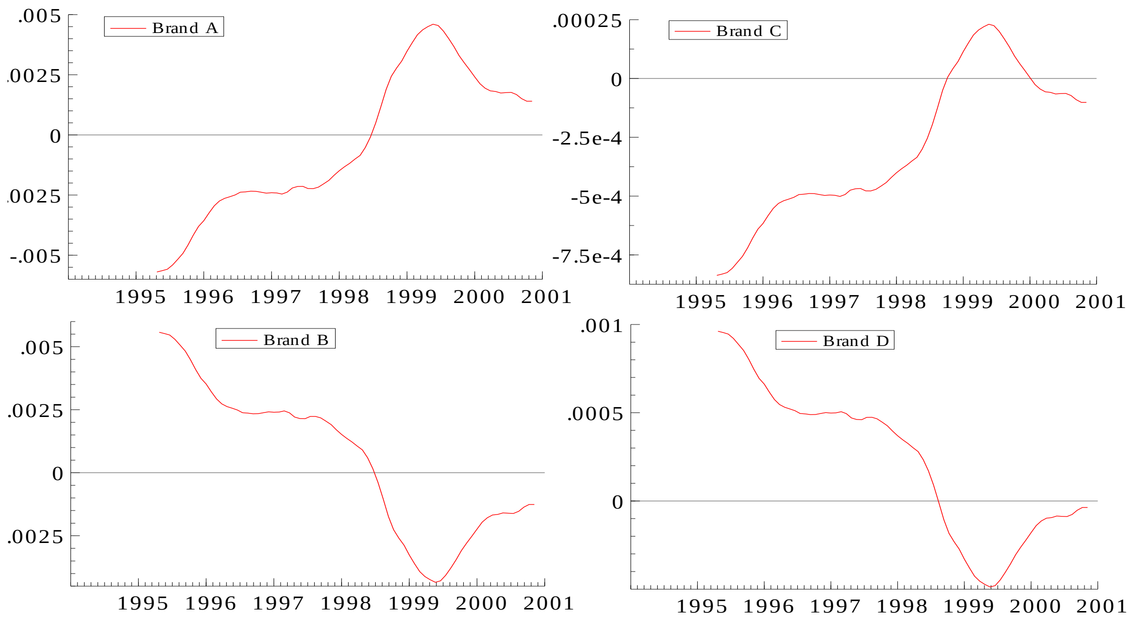
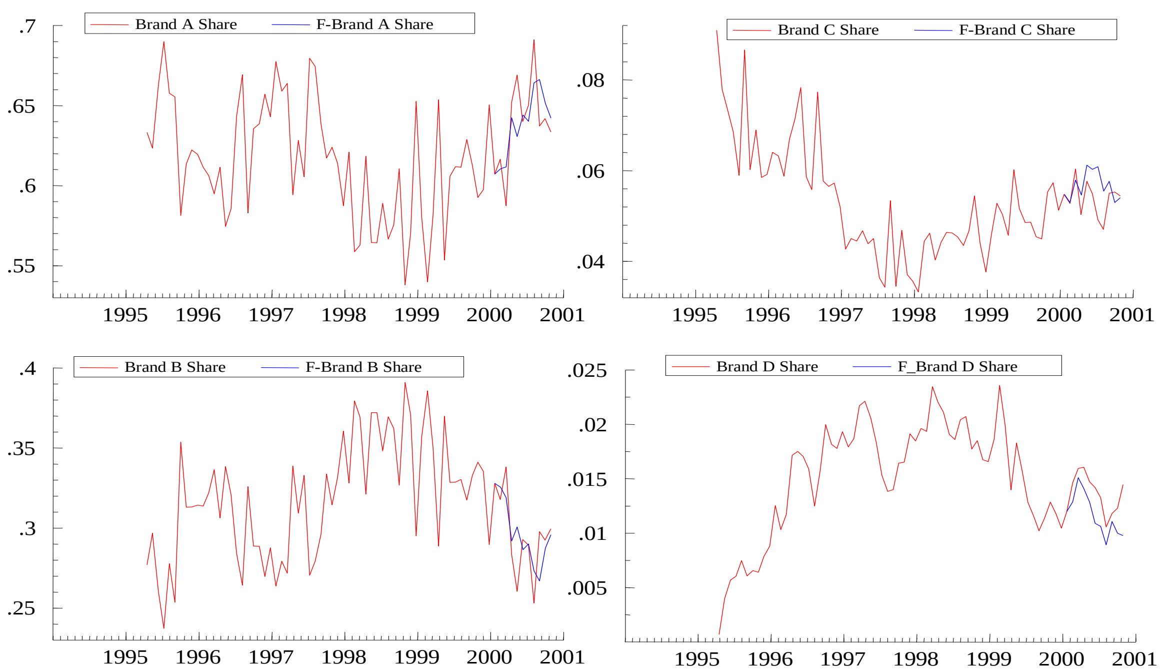


Figure 4: Brand Share Forecasts



[23] These plots depict the underlying stochastic trend present in the first difference of each extracted trend in Figure 2

Table 1: Parameter Estimates for Brand Share Model 2(a)-3(c)[24]

	μ_t	λ_t	$\ln p_{At}$	$\ln p_{Bt}$	$\ln p_{Ct}$	$\ln p_{Dt}$	$\Pr_{At}$	$\Pr_{Bt}$	$\Pr_{Ct}$	$\Pr_{Dt}$	$\ln[w_t/\tilde{p}_t]$
s_{At}	-1.2 (-2.0)	0.001 (1.0)	-0.042 (-0.8)	0.08 (1.0)	0.15 (2.5)	0.03 (0.4)	0.21 (5.0)	-0.06 (-2.0)	-0.05 (-0.5)	0.119 (2.1)	0.14 (3.0)
s_{Bt}	1.76 (3.1)	-0.001 (-0.5)	-0.026 (-0.6)	-0.08 (-1.2)	-0.1 (-1.4)	-0.01 (-0.2)	-0.17 (-4.3)	0.07 (2.1)	0.07 (0.8)	-0.104 (-2.0)	-0.12 (-2.7)
s_{Ct}	0.36 (3.2)	-0.0001 (-0.25)	-0.01 (-1.0)	-0.008 (-1.0)	-0.08 (-7.7)	-0.01 (-0.7)	-0.03 (-4.4)	-0.02 (-3.1)	-0.03 (-1.0)	-0.017 (-2.0)	-0.012 (-1.5)
s_{Dt}	0.08 (2.4)	0.0001 (0.05)	-0.006 (-1.7)	0.008 (1.0)	0.03 (1.9)	-0.01 (-1.7)	-0.01 (-2.7)	0.01 (3.17)	0.01 (0.8)	0.002 (2.0)	-0.008 (-2.2)

Table 2: Brand Share Model (3) compared to two Nested Rival Models[25]

	Model 1				Model 2				Model 3			
	Deterministic Trend and Deterministic Seasonality				Stochastic Trend (μ and λ) Deterministic Seasonality				Stochastic Trend (μ and λ) Stochastic Seasonality			
	S_1	S_2	S_3	S_4	S_1	S_2	S_3	S_4	S_1	S_2	S_3	S_4
R^2	0.46	0.50	0.78	0.78	0.62	0.64	0.90	0.91	0.66	0.70	0.90	0.91
R_s^2	0.42	0.39	0.38	-0.24	0.58	0.56	0.68	0.45	0.63	0.62	0.72	0.48
DW	1.3	1.3	0.75	1.2	2.0	2.1	1.7	1.8	2.0	2.1	1.8	1.8
Q	20.1*	18.0*	57.4*	12.8*	6.2	3.9	12.8*	3.3	6.2	3.8	11.8	3.3
H	0.80	0.75	0.44	0.80	0.63	0.54	0.50	0.75	0.72	0.60	0.60	0.75
N	0.50	0.35	0.30	2.1	4.2	5.6	0.40	2.1	0.15	0.30	0.30	1.1
ACR	1.1	1.2	1.1	1.3	1.05	1.1	1.03	1.07	-	-	-	-

R^2 Coefficient of determination. Appropriate for stationary data with no seasonal component.

R_s^2 R^2 based on differences around the seasonal mean, defined as 1 minus the ratio of the sum of squares for the fitted model to the sum of squares of a random walk with drift and fixed seasonals. A negative statistic implies that the proposed model fit is inferior to a simple random walk with drift and fixed seasonal.

Q(p,q) Box-Ljung statistic for autocorrelation based on the first p autocorrelations distributed as χ^2 with q degrees of freedom – where q is set equal to p+1 minus the estimated number of parameters

H(h) Heteroscedasticity test statistic, equal to the ratio of the squares of the last and first h residuals. It is centered around unity and has an F-distribution with (h,h) degrees of freedom. A high value denotes an increase in error variance over time whereas a low value denotes a decrease.

N(2) Doornik-Hansen normality test, distributed as $\chi^2(2)$ when the model is correctly specified.

ACR Akaike Information Criterion Ratio

[24] The parameter estimates and significance levels for the level and slope factors are given for the end of state. An insignificant t-ratio here does not mean that this was the case over the entire sample.

Table 3: Estimated Covariance and Factor Loading Matrices of the Component Disturbances

Component		Covariance Matrix(Σ^2)				Factor Loading Matrix		
Level	S_A	1.1×10^{-4}	-0.90	-0.62	S_A	1.0000	0.0000	0.0000
	S_B	-8.7×10^{-5}	8.9×10^{-4}	0.18	S_B	-0.8021	1.0000	0.0000
	S_C	-2.6×10^{-5}	7.0×10^{-6}	1.7×10^{-4}	S_C	-0.2428	-0.7252	1.0000
Slope	S_A	7.3×10^{-5}	-1.0	1.0	S_A	1.0000	0.0000	0.0000
	S_B	-6.9×10^{-7}	6.5×10^{-6}	-1.0	S_B	-0.9449	1.0000	0.0000
	S_C	7.2×10^{-8}	-6.8×10^{-8}	7.2×10^{-7}	S_C	0.0989	-0.0002	1.0000
Seasonal	S_A	4.0×10^{-5}	-1.0	-0.25	S_A	1.0000	0.0000	0.0000
	S_B	-4.0×10^{-5}	4.0×10^{-5}	0.16	S_B	-1.0003	1.0000	0.0000
	S_C	-9.9×10^{-7}	6.4×10^{-7}	3.8×10^{-6}	S_C	-0.0249	-1.0434	1.0000

Notes:

1. The covariance matrix Σ^2 for each component can be decomposed, using a Cholesky decomposition, into $\Sigma^2 = \Theta D \Theta'$ where Θ is an Nxp lower triangular matrix with unity values on the leading diagonal and D is a diagonal matrix. The Θ matrix is the factor-loading matrix for each component presented in Table 1in its unrestricted form: that is, it has full rank and we have not imposed any common factors. To impose one common factor and test for co-integration between the level components as in the text, we reduce the rank of the level variance matrix to unity. This amounts to a restriction on the D matrix, which removes the two right hand columns of the factor load matrix.
2. The presence of a random walk or non-stationary component in the brand share data can be evaluated using the tests devised by Nyblom and Harvey (2000). In contrast to the standard ADF test, the null hypothesis is one of stationarity and is rejected if the level covariance matrix $\Sigma_\eta \neq 0$. This is so, if:

   $\zeta_n = tr\left[\mathbf{S}^{-1}\mathbf{C}\right] >$ the relevant critical value

   where:

   $\mathbf{C} = T^{-2}\sum_{i=1}^{T}\left[\sum_{t=1}^{i}(\mathbf{s}_t - \bar{\mathbf{s}})\right]\left[\sum_{t=1}^{i}(\mathbf{s}_t - \bar{\mathbf{s}})\right]'$ cumulation from $i = 1\ldots$T of the squared partial sum of residuals

   $\mathbf{S} = T^{-1}\sum_{t=1}^{T}(\mathbf{s}_t - \bar{\mathbf{s}})\,(\mathbf{s}_t - \bar{\mathbf{s}})'$ = variance of residuals

   $\mathbf{s_t}$ **=** vector containing N time series of shares.
   $\bar{\mathbf{s}}$ = mean share vector.
   tr = the trace calculated from the covariance matrix

   Given we have a local linear trend model with seasonality, we follow Moosa et al (2002) and use the de-seasonalised shares with $(\mathbf{s}_t - \bar{\mathbf{s}})$ calculated as the residuals from a set of OLS regressions of each of the three brand shares on a constant and time trend. The value of ζ_n is 0.638 and with a critical value given in Nyblom and Harvey (2000) of 0.332 at the 5% level for n=3 we conclude that the shares do contain stochastic trends.

[25] A * indicates rejection of the relevant statistic at the 95% confidence level.

Table 4: Elasticities[26]

Conditional Price Elasticities

	Price $_A$	Price$_B$	Price$_C$	Price$_D$
A	-1.25*	-0.22	0.20*	0.03
B	-0.39	-1.09*	-0.25	0.005
C	-0.37	0.19	-2.47*	-0.17
D	0.20*	0.008	2.11*	-1.62*

Conditional Promotional Elasticities

	Pr$_A$	Pr$_B$	Pr$_C$	Pr$_D$
A	0.06*	-0.01*	-0.004	0.012*
B	-0.10*	0.04*	0.004	-0.005*
C	-0.10	-0.06*	-0.01	-0.006*
D	-0.12*	0.10*	0.012	0.004*

Uncompensated Own Price

$$\varepsilon_{ii}^{u} = \left[\frac{\gamma_{ii} - \beta_i \left\{ s_{it} - \beta_i \ln(w/P)_t \right\}}{s_{it}} \right] - 1$$

Own Promotions

$$\varepsilon_{ii}^{X} = \left[\frac{\varphi_{ii} - \beta_i \left(\sum_i \varphi_{ij} \ln p_{it} \right)}{(s_{it} / X_{it})} \right]$$

Uncompensated Cross Price

$$\varepsilon_{ij}^{u} = \left[\frac{\gamma_{ij} - \beta_i \left\{ s_{jt} - \beta_j \ln(w/P)_t \right\}}{s_{it}} \right]$$

Cross Promotions

$$\varepsilon_{ij}^{X} = \left[\frac{\varphi_{ij} - \beta_i \left(\sum_i \varphi_{ij} \ln p_{it} \right)}{(s_{it} / X_{jt})} \right]$$

Segment Expenditure Elasticities[27]

	S$_A$	S$_B$	S$_C$	S$_D$
ε_w	1.23 (3.0)	0.62 (2.7)	0.77 (1.5)	0.46 (2.2)

$$\varepsilon_w = \frac{\beta_i}{s_{it}} - 1$$

[26] All calculations are evaluated at the mean values of share, price, promotions and segment expenditure. Price and promotional elasticities involve both the estimated coefficients and the estimated expenditure elasticities. Consequently the t-ratios attached to the price coefficients in Table 1 cannot be used as a guide to significance. To determine significance, we need to use the distributions associated with the elasticities defined by the elasticity formulae given below. To create this distribution, we adopt a bootstrap approach, which entails drawing *x* random sub-data sets from the full data set available and estimating model 2(a)-3(c) *x* times. This provides *x* estimates of each parameter estimate, from which a set of *x* elasticity estimates can be calculated. We set *x* at 30. The resultant distribution of the estimates is approximately χ^2. To be 90% confident that estimates are significant, the value of each at the 90th percentile of the distribution should be negative. Estimates that satisfy this criterion are indicated with an asterisk.

Table 5: Long-Run Model

	π_{it}	$LTPromo_{At}$	$LTPromo_{Bt}$	$LTPrice_{At}$	$LTPrice_{Bt}$	σ_η^2	R^2	Q	H	N
$\Delta\mu_{At}$	-0.05 (-1.0)	-0.007 (-1.90)	0.006 (2.0)	-0.007 (-0.80)	0.005 (-1.30)	1.1^{-04}	0.17	7.5	0.8	0.67
$\Delta\mu_{Bt}$	0.06 (1.1)	0.007 (1.90)	-0.006 (-2.1)	0.007 (0.96)	-0.005 (1.60)	1.2^{-04}	0.18	7.5	0.7	1.1
$\Delta\mu_{Ct}$	-0.003 (-0.4)	0.0008 (0.50)	-0.0007 (-0.52)	0.0001 (0.03)	0.0002 (0.10)	1.4^{-05}	0.18	24.8*	0.4	1.2
$\Delta\mu_{Dt}$	-0.007 (-0.2)	-0.0008 (-0.60)	0.0007 (0.70)	-0.0001 (-0.40)	-0.0002 (-0.51)	1.7^{-05}	0.16	18.9*	0.8	2.2

Table 6: Comparative Forecast Test Statistic

	s_1	s_2	s_3	s_4
λ_{CD}	2.45	2.50	1.96	1.97
λ_{MNL}	1.80	1.79	2.1	1.84

Notes: The test is based on the following:

$H_0 : E(e_{LLTt}^2) = E(e_{At}^2)$
$H_1 : E(e_{LLTt}^2) > E(e_{At}^2)$

The null/alternative hypothesis is that the economic loss associated with the equations of the local linear trend (LLT) dynamic model (e_{LLTt}^2) is equal to/greater than the loss associated with those of the alternative model(s) (e_{At}^2).

The test statistic is constructed on the basis of a loss differential defined for *n* prediction errors as: $d_t = e_{LLTt}^2 - e_{At}^2$. The test statistic is distributed as students t with *n-1* degrees of freedom and defined as $\lambda = \bar{d} \big/ \sqrt{\hat{v}(\bar{d})}$,where $\bar{d} = n^{-1} \sum_{t=1}^{n} d_t$ and $\hat{v}(\bar{d}) = (n-1)^{-1} n^{-1} \sum_{t=1}^{n} (d_t - \bar{d})^2$

Table 7: Descriptive Data Statistics

	s_{At}	s_{Bt}	s_{Ct}	s_{Dt}	$\ln p_{At}$	$\ln p_{Bt}$	$\ln p_{Ct}$	$\ln p_{Dt}$	Pr_{At}	Pr_{Bt}	Pr_{Ct}	Pr_{Dt}	$\ln[w_t / \tilde{p}_t]$
Mean	0.61	0.31	0.06	0.02	1.8	1.1	1.5	2.0	0.19	0.16	0.02	0.02	10.1
S.D	0.04	0.04	0.01	0.01	0.20	0.10	0.07	0.06	0.10	0.10	0.04	0.07	0.10
Min	0.54	0.24	0.03	0.001	1.4	0.90	1.27	1.8	0.03	0.02	0.0	0.0	9.8

[27] Brand A is the luxury good, with an elasticity greater than unity. Brands B-D are necessities.